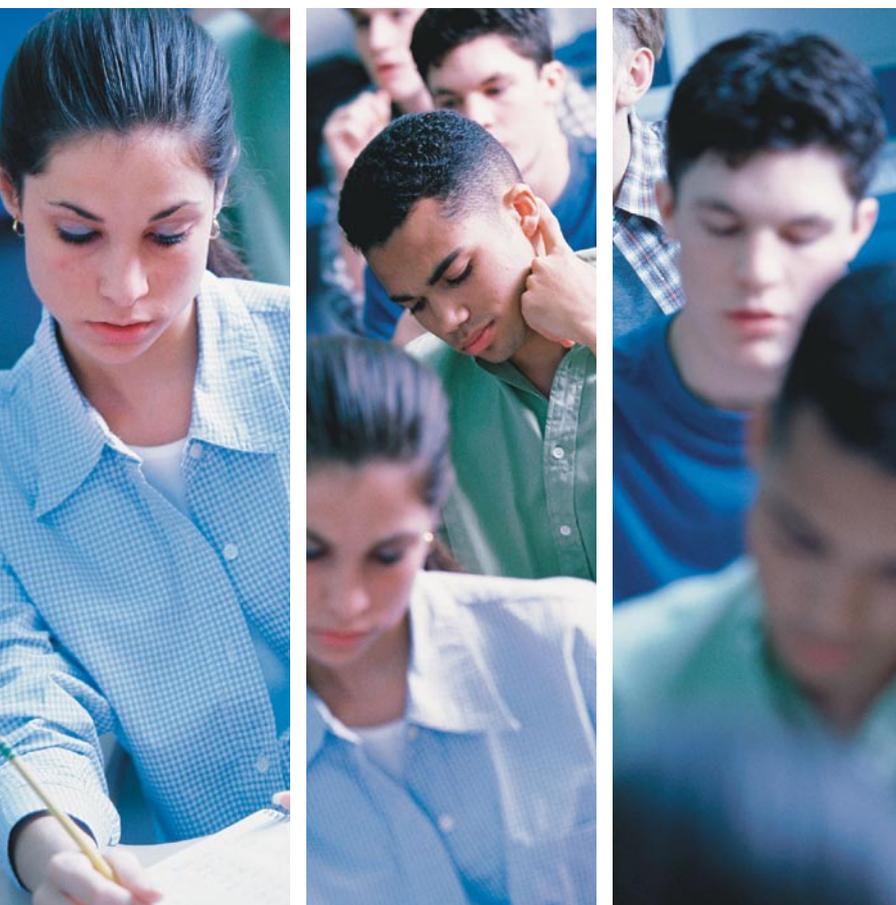


Using a **(BDA)** **Before-During-After** Model to Plan Effective Secondary Mathematics Lessons

Jane Murphy Wilburne and Winnie Peterson



Creating effective mathematics lessons can be a challenge for any teacher. Typically, the biggest hurdle for many teachers is motivating secondary students to want to learn and keeping them interested. The key to motivation is designing lessons that are well organized and centered around student engagement. Whether your district requires well-organized lesson plans or not, it is beneficial to have them. Findings from recent brain research indicate that students are mentally primed to learn during the first fifteen minutes of a typical forty-five-minute class period (Sousa 2001). During the middle fifteen minutes of class time, they experience a mental slump and are more apt to daydream. This research suggests a need to restructure the typical mathematics class from reviewing homework during the first ten minutes of class to getting students' immediate attention by involving them in a problem or activity. One way to design an effective lesson is to use a *before-during-after* (BDA) format.

WHAT IS A BDA?

BDA is a lesson format that divides the class session into three parts, organizing activities so that students can better understand the meaning behind the lesson.

Draw a picture of a rectangular prism and identify three items in a grocery store that are in the shape of a rectangular prism.

Then, suppose a rectangular prism has a volume of 126 cubic cm. What are the possible dimensions of the prism?

Fig. 1 DO NOW



Fig. 2 Taping prisms to the whiteboard

The Before Phase

The *before* phase “hooks” students into the lesson and sets the stage for the main content. The first five to ten minutes of class are important for getting students mentally prepared and involved in the mathematics. The *before* activity can be a quick review of a previous lesson, an opportunity to address common mistakes, or a way to activate students’ prior knowledge. The content in the *before* activity should relate to the *during* phase to assess quickly whether students have the skills needed for the day’s lesson.

For example, when students enter our classroom, a DO NOW is on the overhead (fig. 1). The DO NOW is typically a five- to ten-minute activity to engage the students immediately. The students work individually on the DO NOW while the

teacher takes attendance. The teacher instructs students to pair-share their solutions, and a class discussion on the DO NOW takes place. This brief activity allows the teacher to assess students’ prior knowledge, engages every student in communication with others, involves students in higher-level thinking, and connects mathematics to the real world. The *before* activity has helped prepare students for the day’s lesson.

The During Phase

The core mathematical content of the lesson is embedded in the *during* phase. This phase engages the students in experiments, explorations, or guided discovery of a concept either individually or in small groups.

For the *during* phase that follows the DO NOW in figure 1, groups of four students are each given 20 cm × 20 cm pieces of 1 cm grid paper. Students then cut out equal-size squares from each corner of the larger grid paper, but the side lengths of the smaller square cutouts vary from student to student: 1 cm, 2 cm, 3 cm, . . . , 9 cm. Students fold up the four sides and tape them to form an “open rectangular prism.” Then they find the length, width, height, surface area, and volume of their open prism (table 1). The teacher enters the data for all the prisms on a master table while students fill out their own copy of the same table.

Now it is time to make sense of the data. On the board, the teacher sketches the *x*- and *y*-axes of a graph labeled “height of prism” and “volume,” respectively. Students select a representative from their group to tape a prism onto the graph at its correct position (see fig. 2). For example, the open rectangular prism with height 9 cm and volume 36 cubic cm would be taped at the point (9, 36). A discussion follows about the maximum volume displayed by the graph, and students are asked whether this would be the maximum volume if nonintegral units were allowed. Students pair-share their results to discuss the maximum volumes they found. The *during* phase engages students in building understanding of the concept

Table 1

Prism Worksheet					
Size of Square Cutout	Length of Rectangular Prism	Width of Rectangular Prism	Height of Rectangular Prism	Surface Area of Open Rectangular Prism	Volume of Rectangular Prism
1 cm × 1 cm					
2 cm × 2 cm					
⋮					
9 cm × 9 cm					

Table 2**Teachers' Guide to Developing a *Before-During-After* Lesson**

The <i>Before</i> Phase	<ul style="list-style-type: none"> • Does it relate to today's lesson? • Is it a 5- to 10-minute activity? • Does it grab students' attention? • Does it allow for connections and/or assess prior knowledge? • Do students have the opportunity to share their thinking? • Does it actively engage students?
The <i>During</i> Phase	<ul style="list-style-type: none"> • Is it aligned with academic standards? • Does it meet the course/content objectives? • Does it reflect a problem-solving approach? • Does it promote opportunities for students to communicate their learning? • Are students actively talking, reading, writing, and making sense of the mathematics?
The <i>After</i> Phase	<ul style="list-style-type: none"> • Does it require application of the new knowledge? • Does it assess what the students have learned? • Does it provide opportunity for the students to reflect on their learning and make sense of the mathematics?

and in exploring the mathematical connections in the lesson.

The *After* Phase

The *after* phase focuses on a reflection on the exploration and provides an opportunity for students to make sense of the mathematical meaning of the lesson. It also promotes further reasoning and problem solving through extensions. For example, the students describe how they found the maximum volume of the rectangular prisms in their notebooks. They are also asked to find the surface area of each rectangular prism and write an explanation of why the 1 cm square cutout prism has the maximum surface area. A discussion follows on how an algebraic representation of the volume would be written: $(20 - 2x)(20 - 2x)x = \text{Volume}$, where x is the length of the side of the square that was cut out. Students can enter the data points into their calculators and graph the data. Then they can graph the equation that represents the algebraic representation of the volume. Using the cursor, they can find the maximum volume displayed by the data points and compare their solutions to the nonintegral height of the prism that would hold the maximum volume as represented by the algebraic equation.

Extensions such as the following can be posed: What if the original dimensions were 10 cm \times 10 cm, 30 cm \times 30 cm, or 10 cm \times 20 cm? The *after* phase also provides an opportunity for the students to communicate and summarize their conclusions either in written format or orally, a practice often neglected in mathematics classes.

In addition, the *after* phase can provide oppor-

tunities for the teacher to embed assessments to measure students' conceptual and mathematical understanding. If student explanations seemed confusing, further questions can be posed to help guide them through their understanding of the content, or several additional examples can be shared and discussed.

HOW TO DESIGN A LESSON USING THE BDA MODEL

Preparing a lesson begins with identification of the big ideas you would like students to understand. Aligning these goals with the state and national standards has become important in light of No Child Left Behind. Once the lesson goals are determined, break the lesson into the three parts. The lesson should not be three disjointed activities; each phase should support the others. The questions in **table 2** can guide you in developing the BDA plan.

Figure 3 shows a precalculus lesson designed using a BDA format. The teacher found that his students could visually identify the increasing, decreasing, and constant portions of graphs and functions but had difficulty expressing them in interval format. The lesson focused on helping students communicate interval notation both orally and in writing.

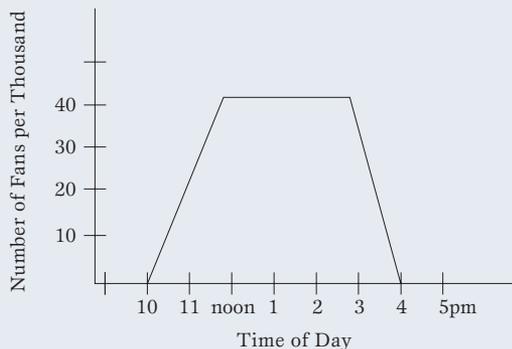
BENEFIT OF USING A BDA MODEL

The BDA model for lesson design benefits the teacher as well as the students because a well-planned and connected lesson provides a consistent flow of concepts throughout the class period. The students are engaged immediately, provided with

Teacher:		Course: Precalculus	Date:
Title	Graphical and Interval Notation		
Essential Questions	<p>How do you express increasing, decreasing, and constant functions represented in a graph in written interval notation?</p> <p>How do you identify the maximum and minimum values in a graph?</p>		
<p>Before Activities: DO NOW!</p> <p>Time: 10 min.</p>	<ul style="list-style-type: none"> Students are given the following inequalities and asked to solve each and represent the graphical solutions in interval notation: $4(3 - x) < -9 \qquad 2x + 3(x + 4) > 11$ Students will pair-share their results and compare their graphical solutions. The teacher will pose questions to the whole class such as: “What approaches did you use to solve the inequalities?” “How does changing the inequality sign affect the solution?” “What does the graphical solution represent?” 		
<p>During Activities</p> <p>Time: 25 min.</p>	<ul style="list-style-type: none"> Students work in pairs to answer the questions on the Football Stadium handout (see fig. 4). The goal is to have students state the range, or intervals of time, where the number of fans was increasing, decreasing, or constant. In groups of four, students share their answers to the questions on the handout. One group will be selected to share its results with the class. The class discusses the increasing, decreasing, and constant intervals over time. The discussion should also focus on the maximum and minimum values of the graphs. Students will write three to five sentences in their notebook about the relationships they see between the slope of the graph at various points and where the function is increasing, decreasing, and constant. Example sentences: “The game was approximately three hours long.” “The slope of the graph is positive to indicate the number of fans entering the stadium and negative to indicate the number of fans leaving the stadium.” “During the game, there are approximately 44,000 fans in attendance.” Students will read their sentences to their partners to compare and later discuss their findings as a class. Next, students work in groups of four on the Soup Can problem (see fig. 5) and state the intervals of increasing, decreasing, and constant temperature for the soup. Students will present their graphs and the intervals to the class and explain their solutions. 		
<p>After Activities</p> <p>Time: 10 min.</p>	<ul style="list-style-type: none"> Students write a four- to five-line summary of how to state the increasing, decreasing, and constant intervals of a graph in their notebooks or as a ticket out the door. For example: “The graph has a positive slope when the soup is heating up in the microwave for three minutes. After that, the temperature cools, so the graph starts declining in a negative slope over the 20-minute period while the soup sat on the counter. The graph showed a constant function at the beginning when the soup can was taken out of the cupboard and the soup was mixed with water at room temperature.” Students pair-share their written summaries, and several students are selected to read theirs to the class. 		
Assignment	Students write a function story, draw the graph of the story, and state the increasing, decreasing and constant intervals for the function.		

Fig. 3 Lesson plan in a BDA format

The following graph depicts the number of fans in the stadium over a period of time at a football game.



Use the graph to answer the following questions:

1. What was the maximum number of fans that attended?
2. At what time did the maximum number of fans first occur?
3. Over what time interval did the number of fans increase?
4. Over what time interval did the number of fans decrease?
5. Over what time interval did the number of fans remain constant?

Fig. 4 Football Stadium handout

opportunities to make sense of the mathematics, and given time to reflect on their learning and summarize or apply their knowledge.

The BDA model helps teachers focus on a strategy-based instructional technique that promotes effective implementation of the lesson. Students are more motivated throughout the class because of the varied activities conducted within one lesson. As research suggests, student learning increases when students are actively engaged and making sense of the mathematics (Hiebert 1999). Students learn new concepts and skills best when they actively construct their own meaning and when they are immediately immersed in a lesson (Sousa 2001, p. 88).

CONCLUSION

The BDA model is already effectively used to engage students in many reading classes. Likewise, mathematics classes designed using this model can promote effective teaching and learning. For mathematics teachers, incorporating a BDA model into a lesson plan can help organize and guide the design of activities to promote and enhance students' mathematical experiences.

The authors would like to thank Louis Quacken-

Eric took a can of soup from a cupboard and poured the soup into a bowl. He then poured water, also at room temperature, into the bowl and stirred the soup and water together. Next, Eric heated the mixture in a microwave oven. After three minutes, he saw the soup boiling and removed the bowl from the microwave and poured some of the contents into a smaller bowl. Eric ate the soup in the smaller bowl, but the rest of the soup cooled down to room temperature in twenty minutes. Finally, Eric placed the original bowl and its contents in the refrigerator, where the soup continued to cool to refrigerator temperature.

Your mission:

- Construct a graph representing the temperature of the soup over time.
- State the intervals when the temperature of the soup is increasing, decreasing, or staying the same.
- Present and explain your graph and intervals to the class.

Fig. 5 Soup Can problem

bush of the mathematics department at John Harris High School in Harrisburg, Pennsylvania, for his help in organizing this article.

BIBLIOGRAPHY

- Hiebert, James. "Relationships between Research and the NCTM Standards." *Journal for Research in Mathematics Education* 30 (January 1999): 3–19.
- National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics*. Reston, VA: NCTM 2000.
- Sousa, David A. *How the Brain Learns*. 2nd ed. Thousand Oaks, CA: Corwin Press, 2001. ∞



JANE MURPHY WILBURNE, jmw41@psu.edu, is an assistant professor of mathematics education at Penn State Harrisburg in Middletown, PA 17057. She is interested in problem solving and student-centered teaching.



WINNIE PETERSON, wpeterso@kutztown.edu, teaches mathematics content and methods to preservice elementary teachers at Kutztown University, PA 19530. Both authors are currently involved in the PA High School Coaching Initiative promoting literacy in mathematics instruction.